Name: ______________________________ Date: ____________________

AP Calculus AB Summer Work

This packet is intended to prepare you for AP Calculus AB by reviewing prerequisite algebra and pre-calculus skills.

It is due on the first full day of school, Wednesday September 4.

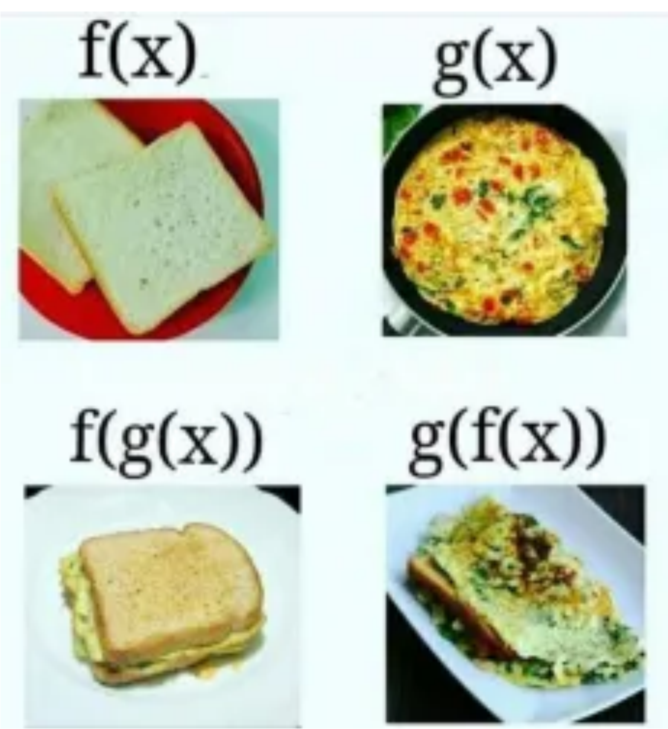
The packet is long, so please start early.

Complete all problems as best as you are able. Show ALL work and use pencil. You may use a calculator. The numbering of the questions may not make sense, so please ignore that.

This will count for the first quiz grade of the year. Grading will be done based on accuracy and completion. You may use notes from pre-calculus to help you.

Have a wonderful and relaxing summer!

If you have questions, please email Miss Johnson. I am excited to be with you all very soon for a wonderful senior year of learning!



The following formulas and identities will help you complete this packet. You are expected to know ALL of these for the course.

LINES

Slope-intercept: $y = mx + b$

Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$

Horizontal line: $y = b$ (slope $= 0$)

Vertical line: $x = a$ (slope $=$ undefined)

Parallel → same slope

Perpendicular → opposite reciprocal slopes

QUADRATICS

Standard: $y = ax^2 + bx + c$

Vertex: $y = a(x - h)^2 + k$

Intercept: $y = a(x - p)(x - q)$

Parabola opens: up if $a > 0$
down if $a < 0$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXPONENTIAL PROPERTIES

$x^a \cdot x^b = x^{a+b}$ $\qquad (xy)^a = x^a y^a$

$\frac{x^a}{x^b} = x^{a-b}$ $\qquad \sqrt[n]{x^m} = x^{m/n}$

$x^0 = 1 \ (x \neq 0)$ $\qquad \left(\frac{x}{y}\right)^a = \frac{x^a}{x^b}$

$x^{-n} = \frac{1}{x^n}$

In general, it is fine to have negative exponents in your answers!

LOGARITHMS

$y = \log_a x$ is equivalent to $a^y = x$

$\log_b(mn) = \log_b m + \log_b n$

$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^p) = p \log_b m$

TRIGONOMETRIC IDENTITIES

$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x} \qquad \tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$

$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$

$\sin(2x) = 2 \sin x \cos x \qquad \cos(2x) = \cos^2 x - \sin^2 x$ **or** $1 - 2\sin^2 x$ **or** $2\cos^2 x - 1$

You are expected to know the general shape, domain, and range of each parent function in the table.

"Parent" functions mean no transformations have been applied. Transformation (shifting, stretching, compressing, or reflecting) may change the domain or range.

Make a sketch of each parent function.

Linear functions (x) **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$	**Quadratic** functions (x^2) **D:** $(-\infty, \infty)$ **R:** $[\mathbf{0}, \infty)$
Odd-degree polynomials (x^n for **odd** $\boldsymbol{n}$) **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$	**Even-degree** polynomials (x^n for **even** $\boldsymbol{n}$) **D:** $(-\infty, \infty)$ **R:** $[\mathbf{0}, \infty)$
Exponential functions (a^x, e^x) **D:** $(-\infty, \infty)$ **R:** $(\mathbf{0}, \infty)$ Horizontal asymptote: $y = 0$	**Logarithmic** functions $(\log_a x, \ln x)$ **D:** $(\mathbf{0}, \infty)$ **R:** $(-\infty, \infty)$ Vertical asymptote: $x = 0$
Sinusoidal functions $(a \sin x, a \cos x)$ **D:** $(-\infty, \infty)$ **R:** $[-\boldsymbol{a}, \boldsymbol{a}]$	**Tangent** function $(\tan x)$ **D:** $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{(2n+1)\pi}{2}, \frac{(2n+3)\pi}{2}\right)$ **R:** $(-\infty, \infty)$ Vertical asymptotes: $x = \frac{(2n+1)\pi}{2}$
Reciprocal functions $\left(\frac{1}{x}\right)$ **D:** $(-\infty, \mathbf{0}) \cup (\mathbf{0}, \infty)$ **R:** $(-\infty, \mathbf{0}) \cup (\mathbf{0}, \infty)$ Vertical asymptote: $x = 0$ Horizontal asymptote: $y = 0$	**Absolute value** functions $(\lvert x \rvert)$ **D:** $(-\infty, \infty)$ **R:** $[\mathbf{0}, \infty)$
Even root functions ($\sqrt{x}$, $\sqrt[n]{x}$ for even n) **D:** $[\mathbf{0}, \infty)$ **R:** $[\mathbf{0}, \infty)$	**Odd root** functions ($\sqrt[3]{x}$, $\sqrt[n]{x}$ for odd $n > 1$) **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$

For #1-8, write an equation for each line in point-slope form.

1. Containing $(4,-1)$ with a slope of $\frac{1}{2}$

____

2. Crossing the x-axis at $x=-3$ and the y-axis at $y=6$

____

3. Containing the points $(-6,-1)$ and $(3,2)$

____

5. Write an equation of a line passing through $(-4,2)$ with a slope of 0.

____

6. Write an equation of a line passing through $(2,8)$ that is parallel to $y=\frac{5}{6}x-1$.

____

8. Write an equation of a line passing through $(6,-7)$ that is perpendicular to $y=-2x-5$.

____

For #9-16, solve each equation for x. Note that some equations with have a specific value, but most will have a solution in terms of other variables. (For example: $x=\frac{a+b}{c}$ may be a solution.)

9. $x^2+3x=8x-6$

____

10. $\frac{2x-5}{x+y}=3-y$

____

11. $3xy + 6x - xz = 12$

12. $A = ax + bx$

14. $r = t - x(z - y)$

16. $\frac{y+2}{4-x} = 4(2 - z)$

For #17-22, solve each quadratic by factoring.

17. $x^2 - 4x - 12 = 0$

18. $x^2 - 6x + 9 = 0$

20. $x^2 - 36 = 0$

____

21. $9x^2 - 1 = 0$

____

22. $4x^2 + 4x + 1 = 0$

____

For #23-27, evaluate the following knowing that $f(x) = 5 - \frac{2x}{3}$ and $g(x) = \frac{1}{2}x^2 + 3x$.

23. $f\left(\frac{1}{2}\right) =$ ______________

25. $f(1) + g(0) =$ ______________

27. $\frac{g(-6)}{f(-6)} =$ ______________

For #28-35, use $f(x) = x^2 - 1$, $g(x) = 3x$, and $h(x) = 5 - x$ to find each composite function.

28. $f(g(x)) =$

____

30. $f(f(4)) =$

____

32. $f\left(g(h(1))\right) =$

____

33. $f\left(g(x-1)\right) =$

____

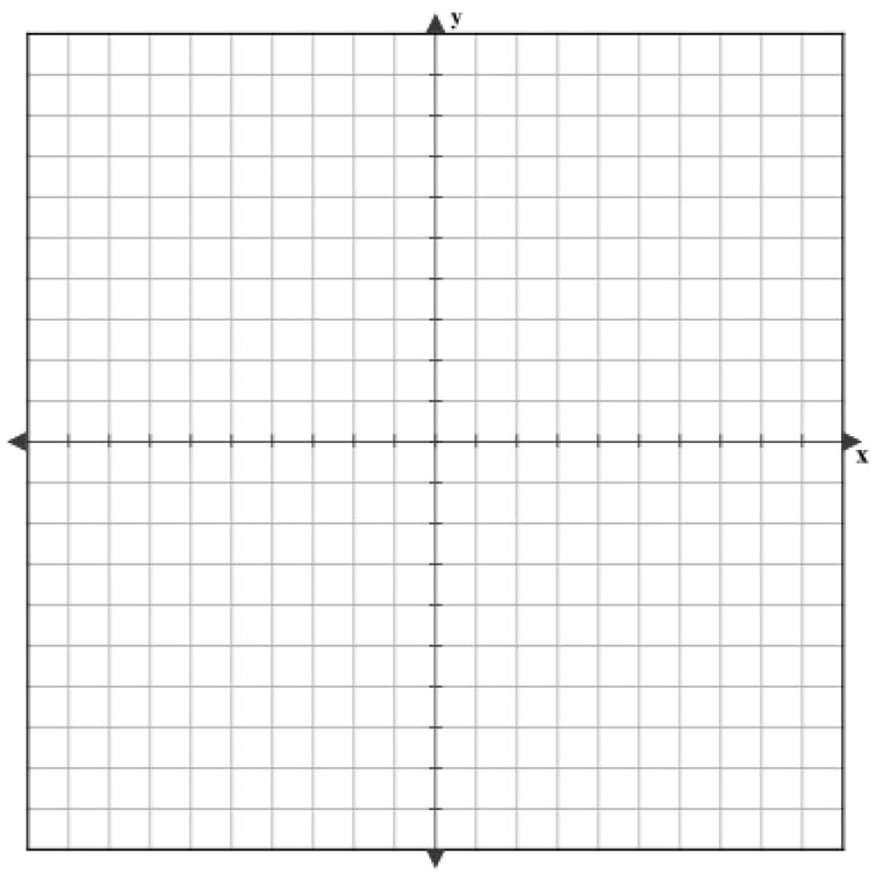
35. $\frac{f(x+h)-f(x)}{h} =$

____

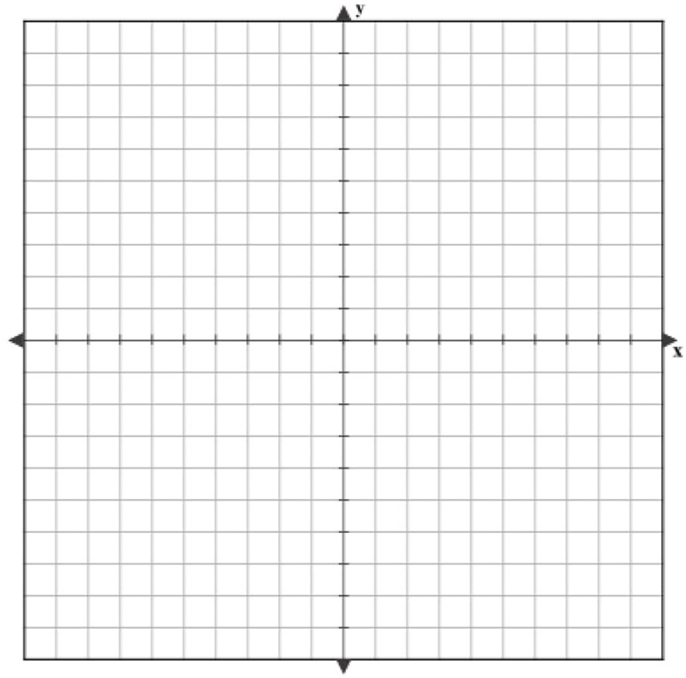
____

For #36-38, graph each piecewise function.

36. $f(x) = \begin{cases} x+3 & ; \ x < 0 \\ -2x+5 & ; \ x \geq 0 \end{cases}$



38. $h(x) = \begin{cases} |x| & ; \ x \leq 1 \\ 2 - |x - 2| & ; \ x > 1 \end{cases}$



For #39-43, solve each exponential equation and round to the nearest thousandth. Some equations can be solved by writing each side as the same base while other will require a logarithm

39. $5^x = \frac{1}{5}$

____

41. $6^{2x-7} = 216$

____

43. $10^{x+5} = 125$

____

For #44-47, simplify each expression without the use of a calculator. The exponential properties on page 2 of this packet will help.

44. $e^{\ln 4} =$ ______________

47. $5 \ln e^3 =$ ______________

For #48-53, solve each exponential or logarithmic equation by hand. Round answers to the nearest thousandth.

48. $e^x = 34$ ____________

50. $e^x - 8 = 51$ ____________

____

52. $\ln(3x - 2) = 2.8$ ____________

53. $2\ln(e^x) = 5$ ____________

For #54-66, find the <u>exact</u> value of the expression using the Unit Circle. To be clear, "exact" answer means no decimals!

54. $\sin 120° =$ ________

55. $\cos \frac{11\pi}{6} =$ ________

56. $\tan 225° =$ ________

57. $\sin\left(-\frac{2\pi}{3}\right) =$ ________

58. $\sin 150° =$ ________

59. $\tan \frac{7\pi}{4} =$ ________

60. $\csc\left(\frac{\pi}{4}\right) =$ ________

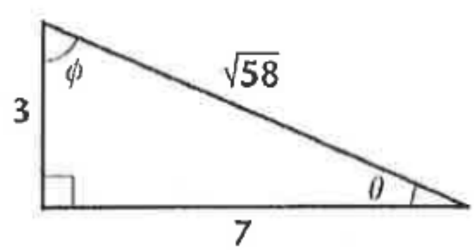
For #67-70, evaluate each trigonometric expression using the right triangle provided. You do <u>NOT</u> need to rationalize the denominator.

67. $\sin\theta =$ ________

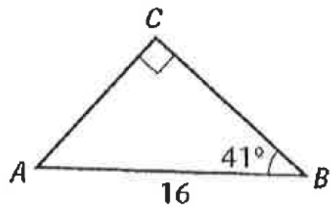
68. $\cos\theta =$ ________

69. $\tan\phi =$ ________

70. $\sec\phi =$ ________



71. Solve the triangle, rounding all angles and sides to the nearest thousandth. ("Solving a triangle" means to find all missing sides and angles.)



$m\angle A =$ ___________

$AC =$ ___________

$CB =$ ___________

For #72-79, evaluate each inverse trigonometric function using the Unit Circle. Write all answer in radians, not degrees. Do not use a calculator.

72. $\sin^{-1}\left(\frac{1}{2}\right) =$ _______

73. $\sin^{-1}(-1) =$ _______

74. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ _______

75. $\tan^{-1}(\sqrt{3}) =$ _______

80. Explain how the graph of $f(x)$ and its inverse, $f^{-1}(x)$, compare.

____

____

For #81-83, find the inverse of each function.

81. $g(x) = \frac{5}{x-2}$

$g^{-1}(x) =$ ____________________

83. $y = \sqrt{4-x} + 1$

$y^{-1} =$ ____________________

TRIGONOMETRIC EQUATIONS

Solve each of the equations for $0 \le x < 2\pi$.

37. $\sin x = -\frac{1}{2}$

38. $2\cos x = \sqrt{3}$

39. $4\sin^2 x = 3$

    **Recall $\sin^2 x = (\sin x)^2$

    **Recall if $x^2 = 25$ then $x = \pm 5$

40. $2\cos^2 x - 1 - \cos x = 0$ *Factor

TRANSFORMATION OF FUNCTIONS

$h(x) = f(x) + c$	Vertical shift c units up	$h(x) = f(x - c)$	Horizontal shift c units right
$h(x) = f(x) - c$	Vertical shift c units down	$h(x) = f(x + c)$	Horizontal shift c units left
$h(x) = -f(x)$	Reflection over the x-axis		

41. Given $f(x) = x^2$ and $g(x) = (x-3)^2 + 1$. How the does the graph of g(x) differ from f(x)?

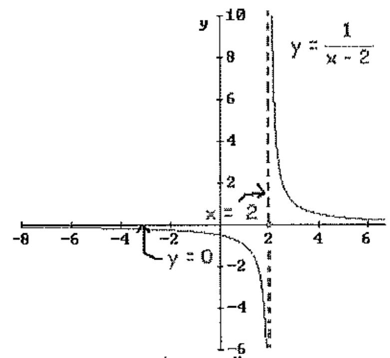
43. If the ordered pair (2, 4) is on the graph of $f(x)$, find one ordered pair that will be on the following functions:

a) $f(x) - 3$ b) $f(x-3)$ c) $2f(x)$ d) $f(x-2)+1$ e) $-f(x)$

VERTICAL ASYMPTOTES

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).
Write a vertical asymptotes as a line in the form x =

Example: Find the vertical asymptote of $y = \frac{1}{x-2}$
Since when $x = 2$ the function is in the form 1/0 then the vertical line $x = 2$ is a vertical asymptote of the function.



44. $f(x) = \frac{1}{x^2}$

46. $f(x) = \frac{2+x}{x^2(1-x)}$

47. $f(x) = \frac{4-x}{x^2-16}$

49. $f(x) = \frac{5x+20}{x^2-16}$

HORIZONTAL ASYMPTOTES

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Example: $y = \frac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Exmaple: $y = \frac{2x^2 + x - 1}{3x^2 + 4}$ (As x becomes very large or very negative the value of this function will approach 2/3). Thus there is a horizontal asymptote at $y = \frac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y = \frac{2x^2 + x - 1}{3x - 3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

Determine all Horizontal Asymptotes.

50. $f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$

53. $f(x) = \frac{(2x-5)^2}{x^2 - x}$

54. $f(x) = \frac{-3x+1}{\sqrt{x^2 + x}}$ * Remember $\sqrt{x^2} = \pm x$

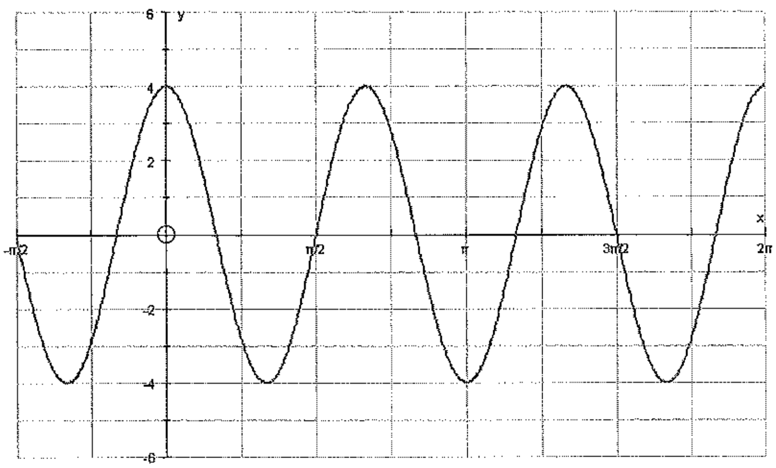
EVEN AND ODD FUNCTIONS

Recall:
Even functions *are functions that are symmetric over the y-axis.*
To determine algebraically we find out if $f(x) = f(-x)$
*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected across the y-axis*)*

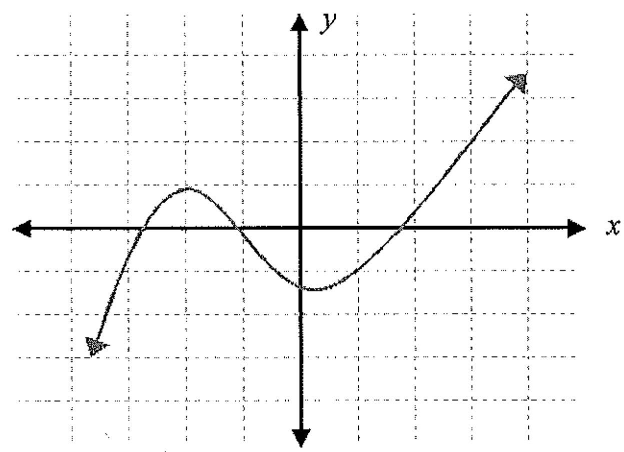
Odd functions *are functions that are symmetric about the origin.*
To determind algebraically we find out if $f(-x) = -f(x)$
*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected over the origin*)*

State whether the following graphs are even, odd or neither, show ALL work.

77. ____________________



78. ____________________



79. ____________________
$f(x) = 2x^4 - 5x^2$

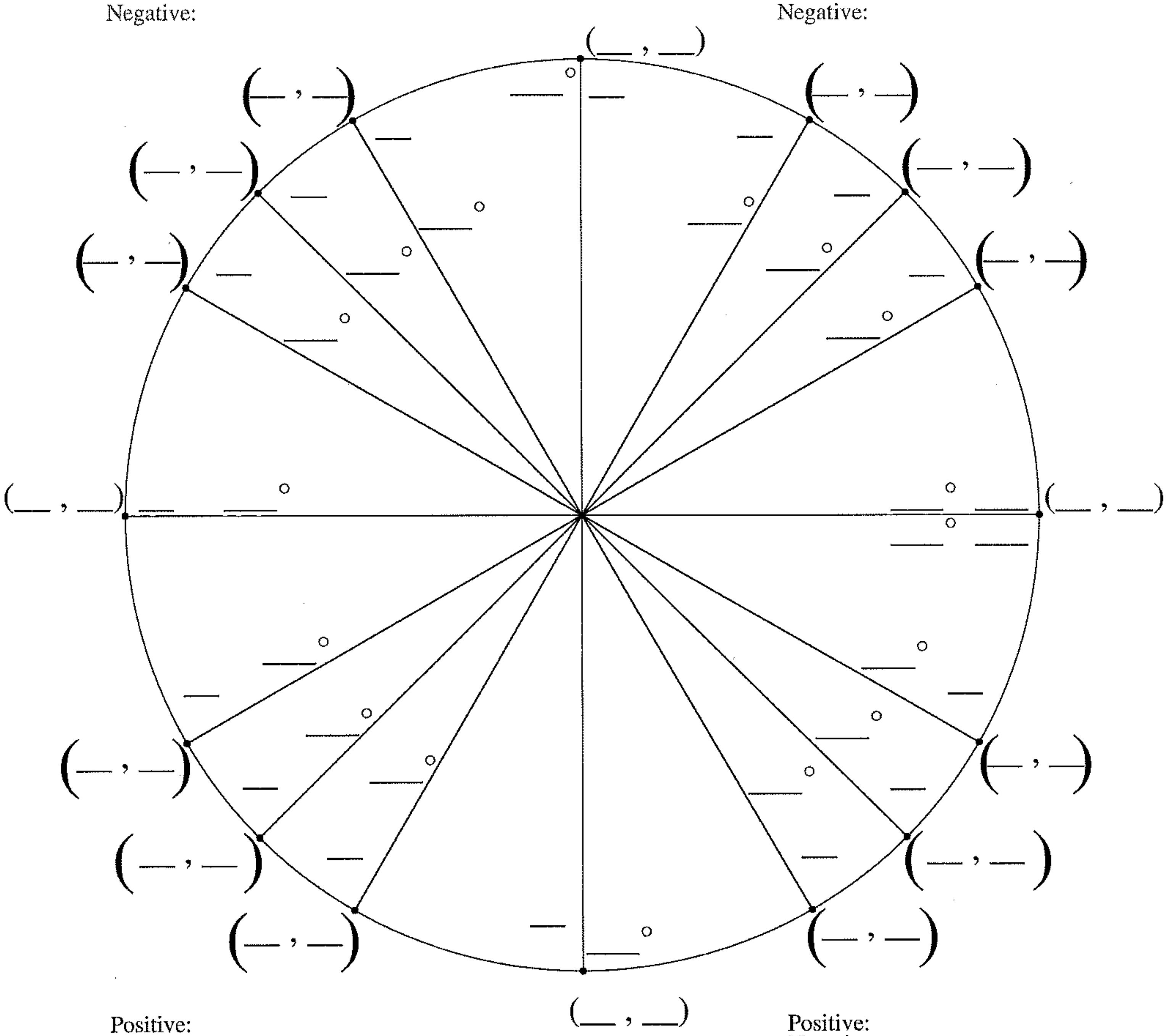
81. ____________________
$h(x) = 2x^2 - 5x + 3$

83. ____________________
$k(x) = \sin x + 4$

Fill in The Unit Circle

Positive:
Negative:

Positive:
Negative:



Positive:
Negative:

Positive:
Negative: